1891MNRAS..52...80I

Delaunay's expressions * for the same being

$$-\frac{1}{a}\left[\frac{15}{16}m+\frac{81}{16}m^2+\frac{5817}{256}m^3\right]\frac{a}{a'}\cos D.$$

$$-0''{\cdot}6136 \quad -0''{\cdot}2479 \quad -0''{\cdot}0832 \quad = -0''{\cdot}9447.$$

$$+\frac{1}{a}\left[\frac{25}{64}m^2-\frac{115}{128}m^3\right]\frac{a}{a'}\cos 3D.$$

$$+0''{\cdot}0191 \quad -0''{\cdot}0033 \quad = +0''{\cdot}0158.$$

The coefficients obtained by using numerical values from the start are $-1''{\cdot}0010$ and $+0''{\cdot}0080$, to which the former pair approximate closely.

15. A comparison of these new values with those given by Hansen cannot be complete owing to the fact that Hansen's numerical values contain the terms due to eccentricity and inclination. But a rough estimate of the uncalculated parts of these series may be made from Delaunay's expressions, since they are carried to a higher degree of approximation for numerical purposes than the terms we have been considering. The whole coefficient of the Parallactic Inequality in the longitude then becomes

$$127''{\cdot}30$$

Newcomb's method of transformation † shows that this coefficient when expressed in Hansen's notation is

	124″·54
Hansen's coefficient is	124·43
Diff.	0·11

The difference is probably partly due to a wrong estimate of the omitted terms in Delaunay's coefficient.

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Secular Perturbations of the Earth's Orbit by Mars.

By R. T. A. Innes.

In Professor Hill's "Theory of *Jupiter* and *Saturn*" he mentions that the terms arising from the fifth powers of the eccentricities and inclination amount to one per cent. of those of the first order in the case of *Mars* and the Earth, and that a computation by Gauss's method would be desirable. I have therefore

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* Delaunay, *Mém. cit.* pp. 922, 923.

† Newcomb, *Mém. cit.*

1891MNRAS..52...80I

made it. To let the comparison with Le Verrier's results be an exact one, I have used the same elements as Le Verrier in "Les Annales de l'Obs. de Paris," *Mémoires*, tome ii. The formulæ used are Professor Hill's, in the appendix to his paper on Gauss's method, published in the Astronomical Papers of the American Ephemeris, where the signification of the symbols will be found.

As a guide to future computers, I give the chief figures involved. This will also lead more easily to the detection of any errors I may have made.

Le Verrier's elements and Professor Hill's formulæ give:

$$\Pi = 51^\circ\ 58'\ 55''{\cdot}25,\ \sin\Pi = [9{\cdot}8964256]$$

$$\Pi' = 284\ \ 55\ \ 5{\cdot}75,\ \tan\Pi' = [n0{\cdot}5744319]$$

$$K = 127\ \ 3\ \ 22{\cdot}70$$

$$K' = K + 53''{\cdot}59$$

$$k = [9{\cdot}9997883]$$

$$k' = [9{\cdot}9999849]$$

$$r = 1 - [8{\cdot}2245450] \cos E$$

$$\tan\frac{v}{2} = [0{\cdot}0072840] \tan\frac{E}{2}$$

$$\nu' = \frac{\sqrt{3}}{32 m_1{}^2}\left\{1 + 2\frac{\lambda^4}{m_1{}^2 m_{11}{}^2} + 4\frac{}{m_1{}^6 m_{11}{}^4 m_{111}{}^2} + \right\}$$

$$\text{Log}\frac{\sqrt{3}}{32} = 8{\cdot}7334106{\cdot}49.$$

$A \quad = r^2 + [9{\cdot}4534182] r \cos(v + 127^\circ\ 3'\ 22''{\cdot}70) + 2{\cdot}3216353$

$B \cos\epsilon = 0{\cdot}2165194{\cdot}5 + [0{\cdot}1826853] r \cos(v + 127^\circ\ 3'\ 22''{\cdot}70)$

$B \sin\epsilon = [0{\cdot}1809850] r \sin(v + 127^\circ\ 4'\ 16''{\cdot}29)$

$C \quad = 0{\cdot}0201929{\cdot}5 = [8{\cdot}3051998]$

$F_1 \quad = [9{\cdot}6379998]\dfrac{B^2 - AC}{3q}$

$F_2 \quad = [n8{\cdot}9713730]\dfrac{B \sin\epsilon}{q}$

$F_3 \quad = F_2 \times [8{\cdot}5095314] r \cos(v + 51^\circ\ 58'\ 55''{\cdot}25)$

$J_1 \quad = 1 - [7{\cdot}0186092] \sin^2(v + 51^\circ\ 58'\ 55''{\cdot}25) - p[9{\cdot}9390298]$

$J_2 \quad = [8{\cdot}7903880] r \sin(v + 127^\circ 3' 22''{\cdot}70) - [7{\cdot}0186092] \sin(v + \Pi) \cos(v + \Pi)$

$J_3 \quad = [8{\cdot}5090779] r \sin(v + 51^\circ\ 58'\ 55''{\cdot}25) - [7{\cdot}1963299] r^2$

To find $A_0{}^{(c)}\ B_0{}^{(c)}\ C_0{}^{(c)}$, etc., etc., I divided E into sixteen parts, and the differences between the sums S and S_1 would seem to show that even division into sixteen parts is not too much.

The values of the functions involved are given in the following tables:—

E	r	v			A	B cos ε	Log B sin ε
°		°	′	″			
0	0·98322954	0	0	0·00	3·1200687	−0·6858181	0·0755814
22½	0·98450612	22	52	14·19	3·0488700	−1·0810004	9·8739366
45	0·98814149	45	41	0·71	3·0196127	−1·2763040	9·2765785
67½	0·99358222	68	23	26·31	3·0367939	−1·2419961	*n* 9·6040431
90	1·00000000	90	57	39·32	3·0978414	−0·9832986	*n* 9·9706386
112½	1·00641778	113	23	5·76	3·1934788	−0·5395969	*n* 0·1232717
135	1·01185851	135	40	31·73	3·3091284	+0·0215597	*n* 0·1826155
157½	1·01549388	157	51	53·67	3·4271406	+0·6147402	*n* 0·1727356
180	1·01677046	180	0	0·00	3·5295059	+1·1496383	*n* 0·0901494
202½	1·01549388	202	8	6·33	3·6006222	+1·5448206	*n* 9·8968887
225	1·01185851	224	19	28·27	3·6296803	+1·7401243	*n* 9·3610636
247½	1·00641778	246	36	54·24	3·6123004	+1·7058160	9·5577875
270	1·00000000	269	2	20·68	3·5511707	+1·4471189	9·9513518
292½	0·99358222	291	36	33·69	3·4556156	+1·0034174	0·1097902
315	0·98814149	314	18	59·29	3·3401649	+0·4422605	0·1708793
337½	0·98450612	337	7	45·81	3·2223515	−0·1509200	0·1607256
S	8·00000000	1260	0	0·00	26·5971730	+1·8552810	
S	8·00000000	1440	0	0·00	26·5971730	+1·8552808	

1891MNRAS..52...80I

1891MNRAS..52...80I

E	p	q^2	r	θ	log m^2	log n^2	log μ
0°	1·0332919	0·4598031	−0·1467474	−28° 4′ 39″·09	9·9941810	9·8022124	9·9504225
22½	1·0095590	0·4636798	−0·1820422	−35 12 32·37	9·9908244	9·8232810	9·9545353
45	0·9998066	0·4650408	−0·1973554	−38 29 8·24	9·9890222	9·8323854	9·9562339
67½	1·0055336	0·4635310	−0·1891672	−36 49 40·95	9·9899544	9·8278233	9·9553892
90	1·0258828	0·4598188	−0·1589229	−30 38 35·56	9·9930618	9·8099969	9·9519690
112½	1·0577619	0·4552393	−0·1127466	−21 32 5·19	9·9965827	9·7812503	9·9461238
135	1·0963118	0·4511423	−0·0596507	−11 21 11·10	9·9990520	9·7451285	9·9383598
157½	1·1356492	0·4482996	−0·0089750	− 1 42 48·42	9·9999784	9·7063832	9·9296824
180	1·1697710	0·4466968	+0·0318289	6 7 11·99	9·9997247	9·6710386	9·9215633
202½	1·1934764	0·4458017	+0·0581844	11 16 21·16	9·9990654	9·6455903	9·9156400
225	1·2031624	0·4451073	+0·0680766	13 15 9·53	9·9987079	9·6352922	9·9132303
247½	1·1973691	0·4445748	+0·0611672	11 54 30·39	9·9989569	9·6423159	9·9148744
270	1·1769926	0·4447342	+0·0381479	7 23 24·16	9·9995985	9·6649389	9·9201483
292½	1·1451409	0·4463320	+0·0009097	0 10 29·25	9·9999998	9·6982042	9·9278176
315	1·1066573	0·4497624	−0·0467645	− 8 55 8·66	9·9994151	9·7357938	9·9362916
337½	1·0673862	0·4546475	−0·0987180	−18 47 6·67	9·9974013	9·7719362	9·9441601
S	8·7918764	3·6221057	−0·4713877	−90 42 56·97	9·9727632	7·8967867	9·4882187
S′	8·7918763	3·6221057	−0·4713880	−90 42 52·80	9·9727633	7·8967844	9·4882228

E	log λ^2	log v'	log $\chi(r)$	$\psi(r)$	log V	$F_1\chi(\tau)$	$F_2\chi(\tau)$
0°	8·3430497	8·8305826	8·7453586	0·0741897	9·2013155	0·02167189	−0·00914170
$22\frac{1}{2}$	8·2919657	8·8228184	8·7132695	0·0696371	9·1650909	0·01831754	−0·00531459
45	8·2660437	8·8196212	8·7000483	0·0678487	9·1502562	0·01707298	−0·00130093
$67\frac{1}{2}$	8·2793701	8·8212164	8·7066194	0·0687315	9·1580297	0·01778349	+0·00281187
90	8·3254405	8·8276703	8·7332790	0·0724400	9·1883721	0·02054752	+0·00698228
$112\frac{1}{2}$	8·3845610	8·8386693	8·7790226	0·0793065	9·2394459	0·02569346	+0·01108019
135	8·4410573	8·8532282	8·8401553	0·0895794	9·3065140	0·03361226	+0·01468882
$157\frac{1}{2}$	8·4872642	8·8694241	8·9090100	0·1028612	9·3809596	0·04428206	+0·01687852
180	8·5205743	8·8845011	8·9739748	0·1173140	9·4504900	0·05643419	+0·01623644
$202\frac{1}{2}$	8·5407042	8·8954511	9·0217167	0·1292960	9·5013472	0·06694892	+0·01162539
225	8·5480973	8·8998932	9·0412263	0·1345584	9·5222568	0·07179179	+0·00354348
$247\frac{1}{2}$	8·5430985	8·8968888	9·0279064	0·1309423	9·5084480	0·06873565	−0·00540872
270	8·5256601	8·8871396	8·9853503	0·1200587	9·4634389	0·05925082	−0·01213458
$292\frac{1}{2}$	8·4956140	8·8729053	8·9238828	0·1059951	9·3976724	0·04721478	−0·01514318
315	8·4533278	8·8570930	8·8564791	0·0925550	9·3245542	0·03607853	−0·01486756
$337\frac{1}{2}$	8·4007099	8·8423608	8·7944432	0·0817742	9·2562021	0·02748428	−0·01252289
S.	7·4232509	0·7597292	8717	439		0·31646000	+0·00400625
S′.	7·4232877	0·7597342	8706	439		0·31646018	+0·00400660

1891MNRAS..52...80I

E	$F_3\chi(\tau)$	J_1	J_2	J_3	$\log aa_1^2 \cos\phi' r q^{-\frac{1}{2}}$	$\frac{a}{r}R_0$.	$\frac{a}{r}S_0$.	$\frac{a}{r}W_0$.
° 0	−0·000178949	+0·1014009	+0·0479185	+0·02349328	0·7764402	0·2258587	−0·0091085	+0·0212508
22½	−0·000044194	+0·1217004	+0·0301830	+0·02916268	0·7724458	0·2138680	−0·0053317	+0·0249943
45	+0·000005543	+0·1301228	+0·0078447	+0·03008817	0·7724555	0·2100120	−0·0011380	+0·0252158
67½	−0·000045662	+0·1253943	−0·0158766	+0·02612875	0·7766056	0·2141949	+0·0031531	+0·0222048
90	−0·000180115	+0·1081083	−0·0375080	+0·01788717	0·7837667	0·2262802	+0·0072616	+0·0156809
112½	−0·000348768	+0·0807171	−0·0537719	+0·006609190	0·7919788	0·2459222	+0·0108250	+0·0049448
135	−0·000476155	+0·0472645	−0·0620825	−0·005962846	0·7992281	0·2719968	+0·0133181	−0·0106056
157½	−0·000480553	+0·0128394	−0·0610082	−0·01794022	0·8042171	0·3017932	+0·0140885	−0·0305409
180	−0·000328671	−0·0172021	−0·0505834	−0·02749054	0·8067070	0·3305172	+0·0125849	−0·0518088
202½	−0·000104440	−0·0381197	−0·0323730	−0·03315980	0·8072503	0·3519508	+0·0087020	−0·0681554
225	+0·000012731	−0·0466033	−0·0092457	−0·03408497	0·8065391	0·3604873	+0·0029870	−0·0725891
247½	−0·000084221	−0·0413425	+0·0151187	−0·03012524	0·8048474	0·3535068	−0·0034062	−0·0625137
270	−0·000304922	−0·0232433	+0·0368684	−0·02188354	0·8018745	0·3326480	−0·0089798	−0·0422440
292½	−0·000466545	+0·0047662	+0·0526579	−0·010614566	0·7971314	0·3034080	−0·0124533	−0·0195472
315	−0·000472020	+0·0382799	+0·0601789	+0·00196604	0·7905904	0·2726627	−0·0133480	−0·0003515
337½	−0·000348180	+0·0721729	+0·0584620	+0·01394311	0·7831251	0·2458188	−0·0119997	+0·0131515
	−0·001922558		−0·0066081	−0·01598724	6·3376016	2·2304629	−0·0035771	−0·1154515
	−0·001922563		−0·0066081	−0·01599610	6·3376016	2·2304627	0·0035776	−0·1154618

1891MNRAS..52...80I

From which

$A_0^{(c)}$ =	+0·2788078	Log = 9·4453050
$\frac{1}{2}A_1^{(c)}$ =	−0·0263243	n 8·4203568
$\frac{1}{2}A_1^{(s)}$ =	−0·0266550	n 8·4257783
$\frac{1}{2}A_2^{(c)}$ =	−0·0003187	n 6·5033400
$B_0^{(c)}$ =	+0·0004472	6·6504737
$\frac{1}{2}B_1^{(c)}$ =	−0·0054333	n 7·7350613
$\frac{1}{2}B_1^{(s)}$ =	+0·0040225	7·6044961
$\frac{1}{2}B_2^{(c)}$ =	+0·0006491	6·8122989
$\frac{1}{2}B_2^{(s)}$ =	+0·0002348	6·3707906
$C_0^{(c)}$ =	−0·0144321	n 8·15933
$\frac{1}{2}C_1^{(c)}$ =	+0·0186839	8·271467
$\frac{1}{2}C_1^{(s)}$ =	+0·0149785	8·17547

And we have

$$\frac{m'n}{1+m} = [9{\cdot}6844068].$$

These give

		Le Verrier's Values.
	″	″
$\left[\frac{de}{dt}\right]_{00}$	−0·018146	−0·01816
$e\left[\frac{d\varpi}{dt}\right]_{00}$	+0·018876	+0·01888
$\left.\frac{dp}{dt}\right]_{00}$	+0·007700	+0·00733
$\left[\frac{dq}{dt}\right]_{00}$	−0·008770	−0·00832
$\left[\frac{dL}{dt}\right]$	−0·270876	−0·2697

The differences are exceedingly small and tend to prove the general correctness of the calculation.

The differences in

$$\left[\frac{dp}{dt}\right]_{00} \text{ and } \left[\frac{dq}{dt}\right]_{00}$$

probably arise from the non-convergency of the function

$$\frac{m'}{2\mu}C$$

in Le Verrier's expansion. The function

$$\frac{m'}{2\mu}G$$

1891MNRAS..52...80I

gives for the

1st order $-0''\!\cdot\!0096363$
3rd „ $-0\cdot0012981$
5th „ $-0\cdot0001487$

from which we may estimate that the terms of higher orders will not exceed $-0''\!\cdot\!00002$.

The equation

$$\left[\frac{da}{dt}\right]_{00}=0$$

gives

$$\sin\phi\tfrac{1}{2}A_1^{(s)}+\cos\phi B_0^{(c)}=0,$$

Substituting the respective numbers I get

$$-0\cdot000447016+0\cdot000447108=0,$$

error—

$$+0\cdot000000092.$$

In making a similar operation on the figures, Professor Hill has given in his example of *Mercury*, perturbated by *Venus*, the difference is more considerable—viz.,

$$-0\cdot00075,$$

probably through a misprint of a figure in the value of R_0 or S_0.

In some parts of the calculations I have used eight decimals, and S and S_1 refer to the original numbers.

Sydney: 1891 *November* 1.

On the Spectrographic Method of determining the Velocity of Stars in the Line of Sight.

By H. C. Vogel, Foreign Associate.

The experiments made at Potsdam in 1887 showed that, as a result of the extremely sensitive photographic methods employed, a sufficiently great dispersion could be made use of to readily detect and measure the displacement of the spectral lines produced by the motion of the stars in the line of sight. It very soon became clear that the measurement of the stellar spectra admitted of a far greater exactness than the direct observations, and that the disturbances of the atmosphere—the chief cause of the difficulties of the direct method—exert their influence in a lesser degree on the photograph. The very numerous measurements on more than two hundred negatives of forty-seven stars,